

# MTH 305: Elementary number theory

## Semester 1, 2020-21

### General information

**Instructor:** Dr. Kashyap Rajeevsarathy

**Office:** AB1-314

**E-mail:** kashyap@iiserb.ac.in

**Venue:**

- Google Classroom: eczighv
- Joining info: [Meet link](#)

**Schedule:** SLOT D (in [Time Table](#))

Day	Timing
Wednesday	10 am - 11 am
Thursday	10 am - 11 am
Friday	10 am - 11 am

### Course structure

#### Topics

- **Foundations:** Principle of mathematical induction (with emphasis on writing a few basic proofs), binomial theorem, countable and uncountable sets, some basic results on countability, countability of  $\mathbb{Z}$ ,  $\mathbb{Q}$  and uncountability of  $\mathbb{R}$ .

- **Divisibility:** Basic properties, division algorithm, GCD, LCM, properties of GCD, relation between GCD and LCM, Euclidean algorithm for finding GCD, Pythagorean triples, linear Diophantine equations, fundamental theorem of arithmetic, Euclid's lemma, existence of infinitely many primes.
- **Modular arithmetic:** Basic properties of congruences, linear congruences, Chinese remainder theorem, Fermat's little theorem, Wilson's theorem. Number theoretic functions: Arithmetic functions (tau, sigma and Mobius) and their properties (specifically multiplicative property of the functions tau, sigma and the Mobius inversion formula), Euler's phi function and its properties, Euler's Theorem, Fermat's little theorem as a corollary of Euler's theorem.
- **Quadratic reciprocity:** Primitive roots (order of an integer modulo  $n$ , primitive roots for primes), quadratic congruences, definition of quadratic residue, Legendre symbol and its properties, quadratic reciprocity law.
- **Continued fractions:** Finite continued fractions, approximation of rational numbers by finite simple continued fractions, solution of linear Diophantine equations using finite continued fractions, infinite continued fractions, unique representation of irrationals as an infinite continued fraction, Pell's equation and its solutions using continued fractions.

## Suggested references

1. David Burton, *Elementary Number Theory (7<sup>th</sup> ed.)*, McGraw Hill Education, 2012.
2. Gareth A. Jones and Josephine M. Jones, *Elementary Number Theory (1<sup>st</sup> ed.)*, Springer Undergraduate Mathematics Series, 1998.
3. Ya. Khinchin, *Continued Fractions (3<sup>rd</sup> ed.)*, Dover, 1997.
4. John Stillwell, *Elements of Number Theory (1<sup>st</sup> ed.)*, Springer, 2003.
5. James Tattersall, *Elementary Number Theory in Nine Chapters (1<sup>st</sup> ed.)*, Cambridge University Press, 1999.

6. Thomas Koshy, *Elementary Number Theory with Applications* (2<sup>nd</sup> ed.), Elsevier, 2007.

## Course policies

### Live interaction sessions

- Reading material and prerecorded video lectures will be posted at the Google Classroom portal every week (details above). It is your responsibility to check the portal for any updates (from my end) and come prepared for the interactive sessions.
- There will be up to 90 minutes of live interaction sessions every week during the scheduled lecture hours. These sessions will be primarily aimed at enhancing your conceptual understanding of the reading material and video lectures posted during the week. Therefore, you are advised to effectively use these sessions to clarify your doubts on the topics being covered.
- If you face any difficulties in participating in the live interaction sessions due to limitations in internet connectivity, data availability/coverage or technology, please contact me immediately. I will try my best to provide additional help or make alternative arrangements for you. Note that all live sessions will be recorded and posted in the Google Classroom for the benefit of the students with poor internet/data connectivity.
- Practice problems will be posted every week, which will be based on the topics covered during the week. It is highly imperative that you try solving these problems on your own, as many problems in the homework assignments and quizzes might be based on them.

### Continuous assessments

Continuous assessment will carry 70% weightage in your final grade. This component of your grade will be based on your **best three performances in four quizzes/assignments** that will be assigned/administered during the course of the semester.

## Assignments

- Every now and then, you will be required to turn in solutions to some select problems from the practice assignments.
- The problems to be turned in and the due dates will be posted on the Google Classroom. So it is your responsibility to regularly check the Google Classroom for any updates.
- If you must miss the due date (for genuine reasons), try turning in your assignment in advance, or write to me seeking an extension.
- The solutions should be turned in via email either as a typed document or as a scanned softcopy of handwritten solutions.
- Problems written should be legible and must clearly indicate the steps used to arrive at the solution.
- While you are encouraged to share and discuss ideas with your classmates, I would strongly caution you against copying solutions verbatim from your classmate/friend.

## Quizzes

- Up to two three quizzes may be administered during the course of the semester.
- All quizzes will be hosted on the Examineer platform developed by the CREATES center. The syllabuses for the quizzes will be announced in the Google Classroom.

## Final assessment

The final assessment will carry 20% weightage in your final grade.

- The schedule for the final assessment will be as per the [academic calendar](#). The final assessment will be comprehensive with more emphasis on topics covered in the second half of the course (i.e. post October 2, 2020).

- The exact nature of this assessment will be announced in due course, as it will largely depend on the prevailing situation at that point in time.

## General policies concerning assessment

- Books, notes, or electronic devices of any kind are strictly prohibited while taking tests (exams and quizzes). It is your ethical (and moral) responsibility to exercise honesty and integrity while taking them.
- When graded tests are returned, please check them carefully for any grading errors. All grading issues should be brought to my attention as soon as possible. Note that your scores are not renegotiable after the final grades are submitted.
- Do not make plans that might prevent you from taking any scheduled exam or quiz. If you have a justifiable reason for missing a scheduled test, you must contact me in advance to make an alternative arrangement.
- Strong disciplinary action will be initiated against students indulging in academic malpractices (or misconduct) during quizzes/exams which include any form of cheating, impersonation, copying, etc., as per the prevailing academic norms of the Institute available at: [Circular - Disciplinary actions for various acts of academic malpractices.](#)

## Grading scheme

A total of 100 percentage points will be distributed as follows:

Component	Weightage
Class participation	10%
Continuous assessment	70%
Final assessment	20%